

Measurements, Preparations, and Interpretations in Quantum Theory: A Comment on Meehan (2020)

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Abstract

I address the recent debate between Meehan and Vaidman concerning the claim made by the former for a new problem to quantum mechanics. I argue that while Meehan's incompatibility claim does hold in the situation he presents, it does not genuinely involve considerations that can limit quantum state preparation, nor does it introduce new constraints over possible interpretations of quantum theory.

Does the process of preparation of physical systems in given quantum states raise foundational issues to quantum theory in a way similar to that in which the measurement process does? Recently, Alexander Meehan ([2020]) offered a positive answer to this question, challenging the accepted view together with the conclusion of an earlier analysis of the preparation process (Wessels [1997]). He argued for a 'control problem' that arises in situations that combine a measurement and a preparation procedure. The problem involves 'our ability to prepare or control quantum states' and is argued to go beyond the measurement problem in its potential of ruling out possible interpretations of quantum mechanics. Meehan's claim was criticized by Vaidman ([2020]), who argued that 'there is no new problem to quantum mechanics'. Vaidman's criticism was very recently addressed in Meehan's reply ([2021]). The aim of this note is to provide a different angle on the issue. In contrast to Vaidman, I accept Meehan's main incompatibility claim. However, I show below that the main conclusions he draws from this incompatibility are unjustified: the situation does not constrain future interpretations of quantum theory, and does not reveal new limits to the possibility of state preparation.

Meehan defines the control problem as a claim of incompatibility of 'individually innocent' assumptions, formulated in a way similar to Maudlin's ([1995]) presentation of the measurement problem in terms of incompatibility of the assumptions of completeness (no hidden variables), linearity (of the temporal evolution) and determinate outcome. According to Meehan:

"The following claims are jointly incompatible:

- (B1) (Preparation): We can successfully prepare quantum states: at least some of our preparation devices are such that, if determinately fed many inputs, they output a non-trivial fraction of those inputs in some specified range of quantum states (here the ‘inputs’ are subsystems, and we define ‘the quantum state of a subsystem’ in the standard way, as its reduced state).
- (B2) (Unitarity): The quantum state of an isolated system always evolves in accord with a deterministic dynamical equation that preserves the inner product, such as the Schrödinger equation [Unitarity].
- (B3) (Determinate Input): It is always determinate whether or not a subsystem has been input into a given (measuring or preparation) device [. . .]
- (B4) (Competent measurement): There is always, from a first-person view, an experienced ‘observed outcome’ of a measurement, even if—from a ‘God’s eye view’—there is no unique determinate outcome. Moreover, our measuring devices are at least somewhat informative and reliable.”

Meehan claims for the incompatibility of these four claims based on a scenario in which an ensemble of quantum systems is prepared in one of two possible states, depending on the outcome of a preceding projective measurement of the z -component of the spin of an electron. As a simplified reconstruction of the argument, consider two cases. The initial spin-state of the electron is $|\uparrow_z\rangle$ in the first case, and $|\uparrow_x\rangle \equiv \frac{1}{\sqrt{2}}(|\uparrow_z\rangle + |\downarrow_z\rangle)$ in the second. The ‘Competent Measurement’ assumption guarantees an observed outcome. This outcome is ‘UP’ in the first case, and either ‘UP’ or ‘DOWN’ in the second case. The experimenter then prepares an ensemble of systems, such that each system is either prepared in a state $|D\rangle$ (if the observed outcome was ‘UP’) or a distinct but non-orthogonal state $|D'\rangle$ (if the observed outcome was ‘DOWN’).¹ According to the ‘Determinate Input’ assumption it is determinate which preparation procedure is being employed, and according to the ‘Preparation’ assumption some preparations succeed and lead to the desired states. At this point we compare an instance of the first case with an instance of the second case in which the initial outcome is ‘DOWN’. The inner product of the initial states is $\langle\uparrow_z|\uparrow_x\rangle = \frac{1}{\sqrt{2}}$. Yet, the systems would be prepared in the state $|D\rangle \times |D\rangle \times \dots |D\rangle$ in the first case and $|D'\rangle \times |D'\rangle \times \dots |D'\rangle$ in the second case, such that the inner product of the final states of the prepared systems decreases with the size of the prepared ensemble and can be arbitrarily small. A violation of ‘unitarity’ (B2) therefore follows from the other three assumptions, which leads Meehan to conclude that the four assumptions are not jointly compatible.

Vaidman objects to the incompatibility claim, and presents in his comment a situation in

¹For simplicity I assume here that the experimenter aims to prepare pure states. This assumption does not constrain the validity of the discussion below. Note that using the deterministic preparation procedure described by Vaidman ([2020]), the prepared states can result as pure states even if the subsystems are initially entangled with other systems. In his footnote 7 Meehan describes the preparation of a mixed state using a Stern-Gerlach apparatus. Note that also in this case, the prepared states would be pure if the spatial degrees of freedom are taken into account.

which all assumptions hold. In response to this criticism, Meehan emphasizes that the ‘apparent innocence’ of the individual assumptions as well as their alleged incompatibility are not inherent to the assumptions, but have to be understood in the context of the given scenario. For example, Vaidman points out that the ‘determinate input’ assumption (B3) generally fails in standard quantum mechanics due to superpositions. Meehan does not deny this claim, but clarifies that the assumption is still plausible in the given scenario.

The same question, however, should be pointed at (B1): when Meehan questions its validity, he is claiming to raise a new problem about ‘what we can do with quantum states’. However, the described situation does not appear to involve any limitation on the *ability* of the experimenter to prepare states, but rather on her *choice* to pick a preparation procedure according to the outcome of a previous measurement.

It should be noted that no objection is raised here to the claim that (B1)-(B4) above, as presented and understood by Meehan, cannot all hold in the given scenario. The aim of this note is to examine whether this incompatibility does indeed impose new constraints on interpretations of quantum theory, as Meehan claims, and whether there is a lesson to be learned from the situation on the limits of state preparation. The obvious point here is that the situation includes a measurement, so any interpretation would have to choose a way of resolving the measurement problem. The control problem could only add to the measurement problem if the incompatibility of (B1)-(B4) is independent of the solution of the measurement problem.

To examine if this is the case, let us first note that the quantum-theoretical concept of linearity is a very strong one and in conflict with our intuitive concept of determinacy. Given a choice between chocolate and strawberry ice cream, conditioning our choice on the outcome of a quantum measurement would lead to a contradiction between the quantum-theoretical assumption of linearity, and the apparently innocent conviction that we are determinately eating the chocolate ice cream. This fact has to do with the quantum measurement problem; it obviously tells us nothing about the nature of ice cream. Can the control problem tell us something about the nature of state preparation? That depends on whether the apparent indeterminacy of the prepared quantum state is, by analogy to the ice cream example, no more than a consequence of the measurement. Meehan’s answer is negative, and he bases it on possible violations of completeness. Hidden variables could determine measurement outcomes and ice-creams flavours. Quantum state preparation, however, is *prima facie* different, since its product is by definition encoded in the wave function, rather than in any additional variables. The aim of my argument below is to show that this difference is not relevant; any solution to the measurement problem does in fact determine a solution to the control problem.

Every interpretation of quantum theory has either to accept or to reject the linearity postulate.² It is easy to note that a violation of linearity entails a necessary violation of the unitarity

²Actually these two options are not the only ones. It is also possible to adopt an epistemic approach (see Ben-Menahem [2017] for a recent discussion) according to which quantum states do not represent reality, such that there is no fact of the matter whether they evolve linearly or not. This option was not considered by Meehan and will not be considered here as well.

assumption (B2). The control problem, therefore, may only impose new constraints on interpretations that satisfy linearity. Let us examine the possible temporal evolutions of the state in such interpretations. In case the electron is initially in the state $|\uparrow_z\rangle$, the evolution is:

$$|\Psi_\uparrow(t_i)\rangle = |\uparrow_z\rangle|initial\rangle_{\text{lab}}|initial\rangle_{\text{subsystems}} \rightarrow |\Psi_\uparrow(t_f)\rangle = |\uparrow_z\rangle_f|UP\rangle_{\text{lab}}|D\rangle \times \dots |D\rangle. \quad (1)$$

The label ‘lab’ here refers to the measuring device, the preparation device, everything that is initially entangled with them and all physical systems (other than the prepared subsystems) that are influenced by the outcome of the initial measurement.³ $|\uparrow_z\rangle_f$ denotes the final spin-state of the electron (it may be identical to its initial state or different from it). If the electron is initially in the state $|\uparrow_x\rangle$ linearity implies that the temporal evolution would take the form:

$$|\uparrow_x\rangle|initial\rangle_{\text{lab}}|initial\rangle_{\text{subsystems}} \rightarrow \frac{1}{\sqrt{2}}|\Psi_\uparrow(t_f)\rangle + \frac{1}{\sqrt{2}}|\Psi_\downarrow(t_f)\rangle \quad (2)$$

with $|\Psi_\downarrow(t_f)\rangle = |\downarrow_z\rangle_f|DOWN\rangle_{\text{lab}}|D'\rangle \times \dots |D'\rangle$ the final state that follows from the initial spin-state $|\downarrow_z\rangle$.

The important point here is that due to linearity, the preparation procedure failed: the subsystems are neither in the quantum state $|D\rangle$, nor in $|D'\rangle$. These states can be regarded as effective states of the subsystems, operationally suitable for making predictions. Yet, the resulting state is considered in Meehan’s analysis as a failure to prepare the desired states. One question that arises here is whether a successful preparation of an *effective* quantum state should indeed be counted as a failure to prepare a quantum state. This question becomes most relevant in the context of Meehan’s and Vaidman’s discussion of Bohmian mechanics, and it is also addressed towards the end of this comment. At this point however, for the clarity of the argument, let us accept Meehan’s view that (B1) fails in situations such as those described by equation (2).

Thus, in all interpretations of quantum theory that resolve the measurement problem by rejection of linearity, the control problem is resolved by the entailed failure of unitarity. In all other solutions of the measurement problem the protocol will not allow for a determinate preparation of the desired pure states, as an indirect consequence of the measurement. Meehan’s paper briefly reflects on the issue, noting a tension between linearity and the other assumptions.⁴ The important note to add, however, is that this incompatibility of linearity and the preparation assumption holds regardless of the validity of the determinate output assumption (B3), and more importantly, regardless of whether the completeness assumption holds.

Yet, one should be more careful: The contradiction between linearity and the determinacy

³This description makes use of the competent measurement assumption (B4), assuming that when the initial quantum state is an eigenstate of the measured observable, the reading of the measuring device reflects the initial quantum state regardless of whether the completeness assumption is valid or not. A rejection of this assumption (which may possibly be implicit in the toy example portrayed in Meehan [2021] pp. 16-17) deprives the quantum state of its operational meaning that makes quantum theory empirically successful and should be the basis for the interpretational discourse.

⁴See the end of Subsection 4.2.

of prepared quantum states is not a contradiction between Meehan's (B1) and (B2). The above criticism would have been valid if Meehan's (B1) had simply read:

(B1'') (Preparation): We can successfully prepare quantum states: at least some of our preparation devices can output a non-trivial fraction of the inputs in some specified range of quantum states.

This assumption is obviously in tension with linearity (as long as a preparation of an effective quantum state in a given branch of the wavefunction is counted as a failure). Meehan, however, chooses to weaken this assumption by constraining it to cases in which the input is determinate (see B1 above). This choice creates an artificial separation between linear completeness-violating theories that can satisfy the precondition and linear theories that adopt the completeness assumption, and can therefore not satisfy the precondition. This separation, however, is only at the level of the premises of the assumption (B1), not of the conclusion. Linear complete and incomplete interpretations do differ in this respect, but they do not differ in 'the extent to which we can control quantum states of the subsystems that we study in the laboratory' as it seems that Meehan tries to argue. For example, Meehan's scenario leads to a failure in the preparation of a pure quantum state in both Bohmian mechanics and the many worlds interpretation, but (B1) is considered in Meehan ([2020]) to be false in Bohmian mechanics, and true in the many worlds interpretation. This truth, however, is a vacuous truth: it does not hold because the preparation procedure is successful, but simply because the determinacy precondition is not satisfied. Thus, the distinction between linear theories that violate (B1) and linear theories that vacuously satisfy (B1) and violate (B3) is completely dependent on the 'determinate input' assumption (B3), whose validity in this scenario is completely dependent (by the assumed protocol) on the 'determinate output' assumption related to the measurement.

'Is the control problem just a non-standard version of the measurement problem?' asks Meehan. This comment answers in the affirmative. Meehan's answer is negative: 'a candidate future theory might successfully avoid the traditional measurement problem, but end up being ruled out by the control problem'. He bases this answer on interpretations that violate completeness. Indeed, supplementing quantum theory by hidden variables can solve the measurement problem, but does not by itself resolve the incompatibility of (B1)-(B4). However, a rejection of completeness does not by itself constitute an interpretation. An adequate solution to the measurement problem (in Meehan's formulation) would also specify whether it accepts or rejects the assumptions of linearity and determinate outcome. Any such solution, it was shown above, will uniquely determine a resolution of the control problem.

To strengthen this point, let us note that the preparation procedure itself plays no substantial role in the control problem. The only step in Meehan's argument that involves the prepared subsystems is the claim for a violation of unitarity based on the vanishing of the inner product of $|D\rangle \times \dots |D\rangle$ and $|D'\rangle \times \dots |D'\rangle$. Vaidman argues that this is irrelevant, since the orthogonality of the final states stems from the orthogonality of the states which describe the possible

measurement outcomes. Meehan, in his reply, denies that the latter ones are necessarily orthogonal, arguing that Vaidman's claim relies on an implicit assumption of completeness. From the perspective presented here it seems that the origin of the dispute is that Vaidman is referring to the orthogonality of the effective states that describe the measurement outcomes in the two cases, while Meehan is referring to the full quantum state (see a more general comment below on the issue). The important point, however, appears to be that in linear theories the value of the inner product $\langle D | \times \dots \langle D | D' \rangle \times \dots | D' \rangle$ does not reveal whether the theory is unitary or not (the relevant inner product is that of the final states in (1) and (2)). The appearance of this inner product in the argument is therefore misleading.

Meehan's argument was clearly presented in terms of two rooms: a left room, in which the measurement is taking place, and a right room in which the preparation procedure is conducted. The conclusion of this comment is that the incompatibility of (B1)-(B4) is already resolved in the left room, it can therefore teach us nothing about the possibility or credibility of state preparation, that is conducted on the room on the right. This incompatibility therefore presents no new problem to quantum mechanics.

Let us conclude by returning to the issue of quantum state preparation from a broader perspective, aiming to focus on possible valuable lessons to be drawn from the debate. The foundational discourse on quantum theory includes at least two different notions of quantum states. The first notion is operationally defined: a 'book-keeping device' used by physicists to make predictions in numerous kinds of experiments. The second notion is ontological, a description of fundamental reality, or at least one of the promising candidates to constitute a description of certain parts of it. The relation between these two notions is interpretation dependent. In some collapse theories the two notions coincide. Epistemic interpretations deny the validity of the second notion. In theories that satisfy the linearity postulate, which are the main focus here, the operationally defined notion supervenes on the ontic notion through the concept of an effective quantum state, which approximates the component of the overall quantum state that is relevant for making predictions. In the many worlds interpretation the effective state is the relative state specified for a given observer. In Bohmian mechanics the effective state of the system is its conditional state, a dynamical property objectively singled out from the universal quantum state by the positions of the Bohmian particles. Since these positions are taken to describe in Bohmian mechanics the factual state of affairs, the term 'the quantum state of a system' is commonly understood to refer to its effective state (see, for example, Dürr *et al.* [2013] Section 2.5).

Which notion is the relevant one for a meaningful discourse on the limits of quantum state preparation?⁵ Preparation procedures are part of the scientific practice, and their success is most naturally evaluated on the backdrop of their pragmatic goals, primarily associated with the operational notion. In the context of no-collapse theories these goals are related to effective

⁵The question is of course relevant in many other contexts, see for example the discussion in (Lazarovici and Hubert [2019]).

states rather than to the ontic notion of the universal wave function. An experimenter who believes in the many worlds interpretation or in Bohmian mechanics regards most parts of the universal wave function as practically inaccessible, when following a preparation procedure she will not aim to achieve more than a preparation of an effective state. With these interpretations in mind, the assumption that we can successfully prepare full ontic quantum states does not appear to be innocent from the beginning.

One possible conclusion from the debate is therefore that future foundational discussion on state preparation would better focus on the concept of effective states. Indeed, if we replace the quantum state in (B1) with an effective quantum state, the four assumptions (B1)-(B4) become compatible (as in Meehan's analysis of Bohmian mechanics). Put another way, the claim for the incompatibility of (B1)-(B4) is at most a claim about empirically irrelevant components of the wave function in linear theories. This phrasing makes it clear from the start that we should not expect it to have the capacity of strictly ruling out possible interpretations. What it does show, however, is that some notion of an effective state is essential in all interpretations of quantum theory that adopt the linearity postulate.

A different and last point to note is the measurement-free preparation procedure described in detail by Vaidman ([2020], Section 3) (and mentioned more generally as a theoretical possibility by Wessels [1997] and Meehan [2020]). This procedure demonstrates that unitary quantum evolution allows, in principle, for the preparation of any desired state, given a quantum system in an arbitrary unknown state (or even one that is entangled with other systems) and a preparation device with a microscopic part whose initial quantum state is known. The significance of this possibility extends beyond this particular debate, and it should be interesting to see its implications further explored.

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References

- Ben-Menahem, Y. [2017]: ‘The PBR theorem: Whose side is it on?’, *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, **57**, pp. 80 – 88.
- Dürr, D., Goldstein, S. and Zanghì, N. [2013]: *Quantum Physics Without Quantum Philosophy*, Berlin, Heidelberg: Springer.
- Lazarovici, D. and Hubert, M. [2019]: ‘How Quantum Mechanics can consistently describe the use of itself’, *Scientific Reports*, **9**(1), pp. 1–8.
- Maudlin, T. [1995]: ‘Three measurement problems’, *Topoi*, **14**(1), pp. 7–15.
- Meehan, A. [2020]: ‘A New Problem for Quantum Mechanics’, *The British Journal for the Philosophy of Science*, Forthcoming. DOI:10.1093/bjps/axz053.
- Meehan, A. [2021]: ‘Clarifying the New Problem for Quantum Mechanics: Reply to Vaidman’, *Foundations of Physics*, **51**(17).
- Vaidman, L. [2020]: ‘There is No New Problem for Quantum Mechanics’, *Foundations of Physics*, **50**, pp. 1728–1734.
- Wessels, L. [1997]: ‘The preparation problem in quantum mechanics’, in J. Earman and J. Norton (eds), *The cosmos of science: Essays of exploration*, University of Pittsburgh Press, pp. 243–273.